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The influence of screen geometry in diffraction by a circular aperture

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Abstract. An edge-current theory, valid for high frequencies, is derived for the problem of electromagnetic diffraction by a circular aperture in a thin conal screen. Calculations of the axial and aperture field are compared with experimental measurements and with the Rayleigh approximations. For the angle of inclination α greater than 90° it is found that the non-planar screen geometry has little effect on the diffracted field, except for the reflection interaction. For $\alpha < 90°$ the fields in and near the aperture are drastically affected, but elsewhere behind the aperture the field shows little change. The edge-current theory predicts the field accurately in all cases except for the reflection interaction.

1. Introduction

The influence of screen geometry in electromagnetic diffraction by a slit aperture in a non-planar screen has been discussed by Tan (1967), who compared results from an approximate solution of the integral equation with experimental measurements and with Kirchhoff's theory. The non-planar slit aperture is essentially a scalar problem, but the corresponding non-planar circular aperture is a much more difficult vector problem whose exact solution has not been attempted. Braunbek (1959) has derived approximate formulae for the electric and magnetic fields **E** and **H** on the axis of a circular aperture formed by the envelope of a system of half-planes inclined at an angle α to the axis. His method was to assume Kirchhoff boundary values for **E** and **H** in the aperture, and to add corrections by fitting half-plane values over a small annulus on the non-planar screen and in the aperture. Tan (1968) has suggested an edge-current theory for this problem, using concepts generalized from Millar's (1955) method, and has applied it to diffraction by a circular aperture formed by the envelope of a system of wedges. The calculations show good agreement with experimental measurements of the aperture and axial electric fields for several screen geometries. It was found that the diffracted fields are insensitive to changes in the wedge angle and to changes in the angle of inclination of the screen on the image side, provided that the sum of these angles is kept constant. However, the geometry of the screen on the source side has a large effect on the aperture field.

This paper investigates further the effect of screen geometry on diffraction by a circular aperture in a thin conal screen. Comparison with the wedge-shaped screen and with the thin plane screen and with Rayleigh's approximations (which takes no account of the shape and nature of the screen), may indicate the relative importance of screen and edge geometry in the diffraction process. An improvement to the edge-current theory is obtained by including a first-order interaction between the fields diffracted by diametral points on the aperture rim. The order of accuracy of the edge-current theory is up to order $(ka)^{-1/2}$, where k is the wave number and a the aperture radius, so that this approximation theory is valid for high frequencies.

Some experimental measurements are presented for comparison with theory. Comparisons have been made in respect of (i) the electric field intensity and phase along two orthogonal diameters in the aperture, the E- and H-plane diameters, parallel and perpendicular, respectively, to the incident electric field vector, (ii) the electric field intensity along the axis of the aperture. Only one aperture size, $ka = 10$ (corresponding to an aperture diameter of 3·18 wavelengths) was used. The angle of inclination of the non-planar screen, α, was varied, the five angles chosen being 48°, 60°, 90°, 120° and 132°.

2. Edge-current theory

The boundary value problem considered here is that of a plane electromagnetic wave with electric vector (0, 1, 0) $\exp(-ikz+i\omega t)$, normally incident on a circular aperture in a thin, perfectly conducting, conal screen (figure 1). A unique and exact solution is required

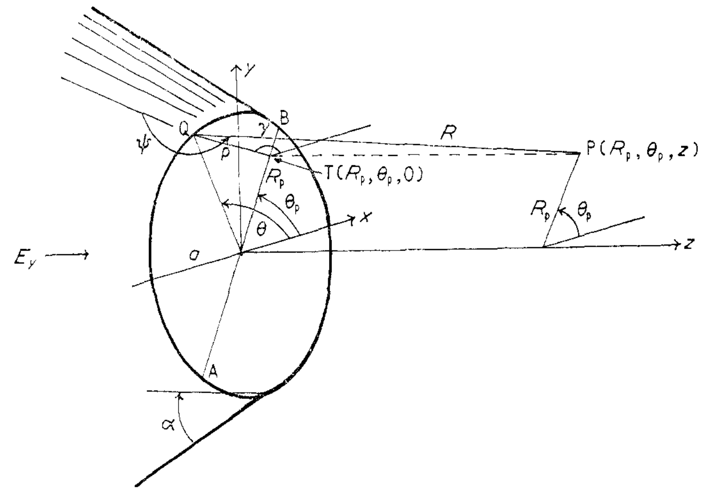

Figure 1. Geometry of diffraction problem.

to satisfy (i) Maxwell's equations, (ii) the boundary condition of vanishing tangential electric field everywhere on the screen, (iii) the edge condition, which governs the order of singularity of the field components at the screen edges, to ensure that the edges are non-radiating (Jones 1964), (iv) the radiation condition that the diffracted far field is an outgoing wave (Baker and Copson 1950). Since an exact solution to this problem is not known, we derive an approximation based on the edge-current theory which has been shown to give good results for planar diffraction problems (Millar 1955, 1956).

The edge-current theory is based on the fact that, in certain regions, the diffracted far field of a half-plane is a diverging cylindrical wave which can be considered to have been radiated by fictitious electric and magnetic currents I_e and I_m on the half-plane edge. If we now consider the aperture diffraction problem in figure 1, and assume that the aperture edge at every point behaves like a tangential half-plane, then the diffracted field of the aperture is given by the combined radiated fields of the currents I_e and I_m on the rim. In particular, the approximate formulae for the total aperture and axial fields, to order $(ka)^{-1/2}$, are given by (Millar 1955, Tan 1968)

$$E_x(R_p, \theta_p, 0) = -\frac{aD_e(\alpha, \alpha+\frac{1}{2}\pi)}{\sqrt{2\pi}} \int_0^{2\pi} \frac{\exp(-ik\rho)}{\rho} \cos(\gamma-\theta)\cos\theta\sin\gamma\, d\theta \tag{1}$$

$$E_y(R_p, \theta_p, 0) = 1+\frac{aD_e(\alpha, \alpha+\frac{1}{2}\pi)}{\sqrt{2\pi}} \int_0^{2\pi} \frac{\exp(-ik\rho)}{\rho} \cos(\gamma-\theta)\cos\theta\cos\gamma\, d\theta \tag{2}$$

$$E_y(0, 0, z) = \exp(-ikz) - \frac{a\exp(-ikR)}{\sqrt{2R}} \left\{D_m(\alpha, \psi)\frac{z}{R} - D_e(\alpha, \psi)\right\} \tag{3}$$

where the asymptotic factors D_e and D_m are

$$\left.\begin{matrix} D_e(\alpha, \psi) \\ D_m(\alpha, \psi) \end{matrix}\right\} = \frac{1}{2\sqrt{2}} [\sec\{\tfrac{1}{2}(\psi+\alpha)\} \mp \sec\{\tfrac{1}{2}(\psi-\alpha)\}] \tag{4}$$

and

$$\psi = \alpha + \tfrac{1}{2}\pi + \tan^{-1}\left(\frac{z}{a}\right)$$
$$\rho = \{R_p{}^2 + a^2 - 2aR_p \cos(\theta - \theta_p)\}^{1/2}$$
$$R = (\rho^2 + z^2)^{1/2}$$
$$\gamma = \theta_p + \pi - \sin^{-1}\left\{\frac{a}{\rho}\sin(\theta - \theta_p)\right\}.$$

Written explicitly, equation (3) becomes

$$E_y(0, 0, z) = \exp(-ikz) + \frac{a}{R}\frac{\exp(-ikR)}{\cos\alpha + \cos\psi}\left(\sin\tfrac{1}{2}\alpha \sin\tfrac{1}{2}\psi - \frac{z}{R}\cos\tfrac{1}{2}\alpha\cos\tfrac{1}{2}\psi\right) \quad (5)$$

which is identical with Braunbek's (1959) result.

Equations (4) for the asymptotic factors are invalid when the point of observation P approaches the geometrical optics shadow region or the reflected ray region, since the cosine terms then tend to zero. The reflected rays will travel close to the aperture plane when α is near $\frac{1}{4}\pi$. Hence, when α is near $\frac{1}{4}\pi$ and P lies in the aperture, D_e and D_m have to be derived in an alternative way. The required results are derived from Sommerfeld's exact solution of the half-plane problem (Born and Wolf 1965). The *E*- and *H*-polarized solutions are

$$\left.\begin{matrix} E_y(r, \phi) \\ H_y(r, \phi) \end{matrix}\right\} = \frac{\exp(-ikr) + \frac{1}{4}i\pi}{\sqrt{\pi}}\left(G[-(2kr)^{1/2}\cos\{\tfrac{1}{2}(\phi-\alpha)\}] \mp G[-(2kr)^{1/2}\cos\{\tfrac{1}{2}(\phi+\alpha)\}]\right) \quad (6)$$

where

$$G(v) = \exp(iv^2)F(v) = \exp(iv^2)\int_v^\infty \exp(-it^2)\,dt$$

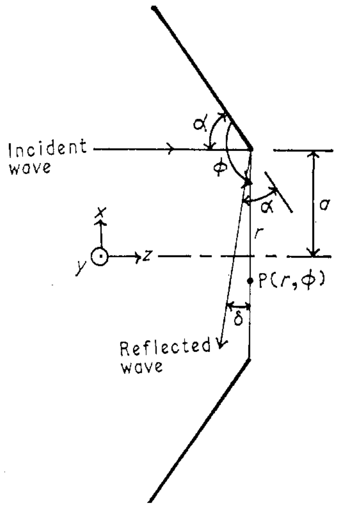

Figure 2. Diffraction in the aperture for $\alpha \gtrsim 45°$.

and r, ϕ are the coordinates of the point of observation relative to the edge of the half-plane (see figure 2). For α greater than, but close to, $\frac{1}{4}\pi$ and $r \sim a$, $\phi = \alpha + \frac{1}{2}\pi - \delta$, we have, for $\delta \to 0$,

$$\left.\begin{matrix} E_y(r \sim a, \alpha + \frac{1}{2}\pi) \\ H_y(r \sim a, \alpha + \frac{1}{2}\pi) \end{matrix}\right\} \simeq \frac{\exp(-ikr + \frac{1}{4}i\pi)}{\sqrt{\pi}}\Bigg[\exp(ikr)F\{-(kr)^{1/2}\}$$
$$\mp \exp\{ikr(\cos\alpha - \sin\alpha)^2\}F\left\{-(kr)^{1/2}(\cos\alpha - \sin\alpha)\left(\frac{a}{r}\right)^{1/2}\right\}\Bigg]. \quad (7)$$

Using the asymptotic expansions for the Fresnel integrals with large and small arguments, the diffracted far fields in the region of the reflected ray are then given by

$$\left.\begin{matrix} E_y{}^{\mathrm{d}}(r \sim a, \alpha+\frac{1}{2}\pi) \\ H_y{}^{\mathrm{d}}(r \sim a, \alpha+\frac{1}{2}\pi) \end{matrix}\right\} \simeq \frac{\exp(-ikr+\frac{1}{4}i\pi)}{\sqrt{\pi}} \left\{\frac{i}{2(kr)^{1/2}} \mp \tfrac{1}{2}\sqrt{\pi}\exp(-\tfrac{1}{4}i\pi) \mp (ka)^{1/2}(\cos\alpha - \sin\alpha)\right\}$$

$$\simeq \frac{\exp(-ikr-\frac{1}{4}i\pi)}{(\pi kr)^{1/2}} \left\{-\tfrac{1}{2} \mp \tfrac{1}{2}(\pi ka)^{1/2}\exp(\tfrac{1}{4}i\pi) \mp ika(\cos\alpha - \sin\alpha)\right\}. \quad (8)$$

The asymptotic factors for this case are then given by the expressions in curly brackets in equation (8)

3. Interaction scheme

An improvement of the edge-current theory can be obtained by including the interaction between the diffracted fields from different points of the aperture edge. As a first approximation, only the interaction between fields from diametrically opposite points of the aperture will be calculated. This is essentially the same type of interaction scheme used by Clemmow (1956) and Karp and Russek (1956). Using Millar's (1956) procedure, the integrals for the diffracted fields in the aperture, from equations (1) and (2),

$$\left.\begin{matrix} E_x{}^{\mathrm{d}}(R_{\mathrm{p}}, \theta_{\mathrm{p}}, 0) \\ E_y{}^{\mathrm{d}}(R_{\mathrm{p}}, \theta_{\mathrm{p}}, 0) \end{matrix}\right\} = \mp \frac{aD_{\mathrm{e}}(\alpha, \alpha+\frac{1}{2}\pi)}{\sqrt{2\pi}} \int_0^{2\pi} \frac{\exp(-ik\rho)}{\rho} \cos(\gamma-\theta)\cos\theta \begin{matrix} \sin\gamma \\ \cos\gamma \end{matrix} \, d\theta \quad (9)$$

are expanded asymptotically by the method of steepest descent to give the terms

$$\left.\begin{matrix} E_x{}^{\mathrm{d}}(R_{\mathrm{p}}, \theta_{\mathrm{p}}, 0) \\ E_y{}^{\mathrm{d}}(R_{\mathrm{p}}, \theta_{\mathrm{p}}, 0) \end{matrix}\right\} = \mp \left\{\begin{matrix} \frac{1}{2}\sin 2\theta_{\mathrm{p}} \\ \cos^2\theta_{\mathrm{p}} \end{matrix}\right\} \frac{aD_{\mathrm{e}}(\alpha, \alpha+\frac{1}{2}\pi)}{(\pi kaR_{\mathrm{p}})^{1/2}}$$

$$\times \left[\frac{\exp\{-ik(a+R_{\mathrm{p}})+\frac{1}{4}i\pi\}}{(a+R_{\mathrm{p}})^{1/2}} + \frac{\exp\{-ik(a-R_{\mathrm{p}})+\frac{1}{4}i\pi\}}{(a-R_{\mathrm{p}})^{1/2}}\right] + \mathrm{O}\left(\frac{1}{ka}\right). \quad (10)$$

The integrand in (9) can be shown (Millar 1956) to have two stationary points A and B on the aperture rim, and lying on the diameter ATB in figure 1. The first two terms of the asymptotic series in (10) correspond to the diffracted waves at T coming from the stationary points A and B. The diffracted wave from A is considered to travel across the aperture and to be incident on the aperture edge at B. This incident wave at B, resolved into components parallel and perpendicular to the edge, is

$$\left.\begin{aligned} E_{\parallel}{}^{\mathrm{d}}(a, \theta_{\mathrm{p}}) &= E_y{}^{\mathrm{d}}\cos\theta_{\mathrm{p}} - E_x{}^{\mathrm{d}}\sin\theta_{\mathrm{p}} = \cos\theta_{\mathrm{p}} \frac{D_{\mathrm{e}}(\alpha, \alpha+\frac{1}{2}\pi)}{(2\pi ka)^{1/2}} \exp(-2ika+\tfrac{1}{4}i\pi) \\ E_{\perp}{}^{\mathrm{d}}(a, \theta_{\mathrm{p}}) &= E_y{}^{\mathrm{d}}\sin\theta_{\mathrm{p}} + E_x{}^{\mathrm{d}}\cos\theta_{\mathrm{p}} = 0. \end{aligned}\right\} \quad (11)$$

This field $E_{\parallel}{}^{\mathrm{d}}$, considered as an E-polarized plane wave incident on the half-plane edge at B, gives rise to the interaction field at the point (R, ψ) relative to the half-plane edge:

$$E_{\parallel}{}^{\mathrm{int}}(R, \psi) = \frac{\exp(-ikR)}{(\pi kR)^{1/2}} D_{\mathrm{e}}(\alpha+\tfrac{1}{2}\pi, \psi) E_{\parallel}{}^{\mathrm{d}}(a, \theta_{\mathrm{p}}) \quad (12)$$

which can be represented by an electric edge current dI_{e}, where (Tan 1968)

$$\beta = \frac{dI_{\mathrm{e}}}{I_{\mathrm{e}}} = i\frac{\exp(-2ika)}{(2\pi ka)^{1/2}} \frac{D_{\mathrm{e}}(\alpha, \alpha+\frac{1}{2}\pi) D_{\mathrm{e}}(\alpha+\frac{1}{2}\pi, \psi)}{D_{\mathrm{e}}(\alpha, \psi)}.$$

With this interaction term, the expressions for the aperture and axial fields should be modified by replacing D_{e} by $D_{\mathrm{e}}(1+\beta)$. To this order of approximation, the magnetic currents do not contribute to the interaction. Comparison with experimental measurements shows that this interaction term improves accuracy considerably in regions where the diffracted field intensity is low, but elsewhere the improvement is small. Higher-order interactions would not be justifiable here, since equations (1) and (2) are accurate to order $(ka)^{-1/2}$ only in the first place.

4. The Rayleigh approximations

The well-known first and second Rayleigh approximations are obtained by assuming that the unknown tangential electric and magnetic fields in the aperture can be replaced by the corresponding incident fields, giving (Jones 1964)

$$\mathbf{E}_1(R_p, \theta_p, z) = \frac{1}{2\pi} \nabla \times \int_A (\mathbf{n}_z \times \mathbf{E}^i) \frac{\exp(-iks)}{s} dA \tag{13}$$

$$\mathbf{E}_2(R_p, \theta_p, z) = \frac{1}{2\pi ik} \nabla \times \nabla \times \int_A (\mathbf{n}_z \times \mathbf{H}^i) \frac{\exp(-iks)}{s} dA \tag{14}$$

$$s = \{z^2 + R_p{}^2 + r_A{}^2 - 2r_A R_p \cos(\theta_A - \theta_p)\}^{1/2}$$

where s is the distance between the point of observation $P(R_p, \theta_p, z)$ and the point of integration in the aperture $(r_A, \theta_A, 0)$, and $\mathbf{n}_z$ is a unit vector along the z axis and normal to the aperture plane. With a plane wave, $(0, 1, 0)\exp(-ikz)$, normally incident on a circular aperture of radius a (figure 1), the axial fields from (13) and (14) are, by direct integration (Jones 1964, p. 637),

$$E_{1y}(0, 0, z) = \exp(-ikz) - \frac{z}{(a^2+z^2)^{1/2}} \exp\{-ik(a^2+z^2)^{1/2}\} \tag{15}$$

$$E_{2y}(0, 0, z) = \exp(-ikz) - \frac{1}{2}\left\{1 + \frac{z^2}{a^2+z^2} + \frac{ia^2}{k(a^2+z^2)^{3/2}}\right\} \exp\{-ik(a^2+z^2)^{1/2}\}. \tag{16}$$

To evaluate the field at off-axis points, (13) and (14) can be transformed into line integrals, which are more convenient for computation purposes. For comparison with the edge-current theory in figures 3 and 4, the second Rayleigh integral is computed from a formula derived from (14) by Marchand and Wolf (1962) and Wu (1966):

$$\begin{aligned} E_{2y}(R_p, \theta_p, z) = \Bigg[\exp(-ikz) &+ \frac{a}{2\pi ik} \int_0^\pi \frac{\exp(-ikR')}{(R')^2} \left\{\frac{1}{R'} - ik\left(\frac{2z^2}{t^2}+1\right)\right\} (a - R_p \cos\theta)\, d\theta \\ &- \frac{a\cos 2\theta_p}{2\pi ik} \int_0^\pi \frac{\exp(-ikR')}{R'} \left(1 - \frac{2a^2 \sin^2\theta}{t^2}\right) \left(\frac{2}{t^2} + \frac{1}{(R')^2} + \frac{ik}{R}\right) \\ &\times (a - R_p\cos\theta)\, d\theta\Bigg\} \end{aligned} \tag{17}$$

$$R' = (z^2 + a^2 + R_p{}^2 - 2aR_p \cos\theta)^{1/2}$$

$$t = (a^2 + R_p{}^2 - 2aR_p\cos\theta)^{1/2}.$$

5. Results and conclusions

Microwave measurements at 3·2 cm wavelengths have been made inside an anechoic room on several conal screens constructed from aluminium sheets of 0·7 mm thickness. The accuracy of the geometry of the cone near the aperture was to within 1 mm, while the size of the screen was about 70 wavelengths, certainly large enough to simulate an infinite screen for measurements near the aperture. The microwave anechoic room was lined with absorbers, whose reflectivity was about −50 dB at normal incidence. The probe used was a slot-fed electric dipole 8 mm long, connected to a coaxial line of 2 mm outer diameter. The electric field intensity measurements used a radio-frequency substitution method with a calibrated reference attenuator, while the phase measurements used a homodyne method (Robertson 1949). The overall accuracy of the measurements was estimated to be 0·2 dB for the intensity measurements, and 3° for the phase measurements.

Figures 3 and 4 show a comparison of theory and measurement of the H-plane and E-plane aperture electric field intensity and phase for angles of inclination α varying from

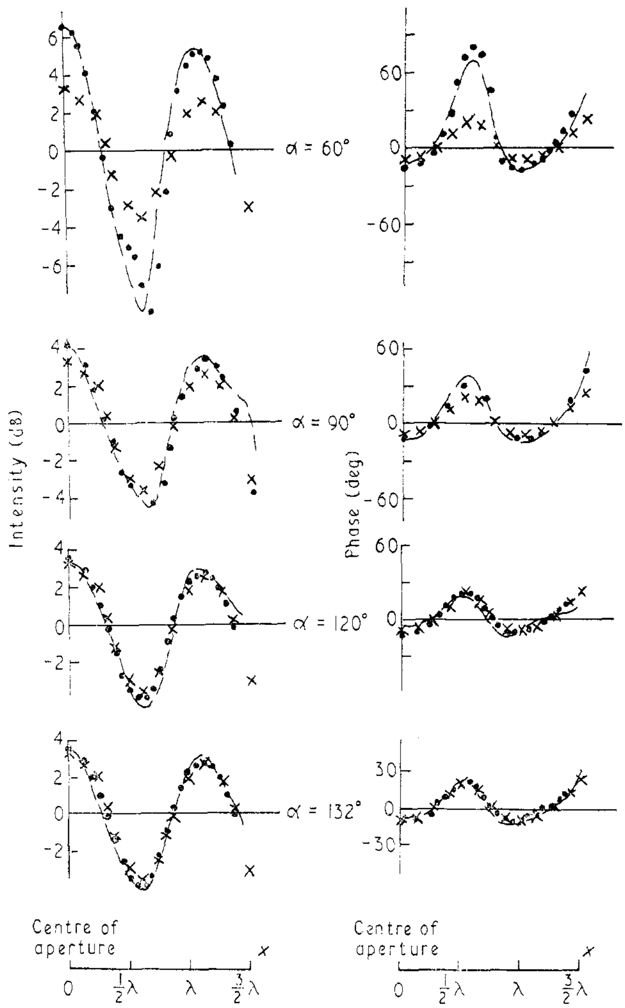

Figure 3. H-plane aperture field $E_y{}^2(x, 0, 0)$ of circular aperture with inclination angle α, $ka = 10$ ($\lambda = \pi a/5$): – – – – experiment; • • • edge-current theory (equation (2)); × × × second Rayleigh integral (equation (17)).

48° to 132°. The results for $\alpha = 48°$ H-plane measurements have already been reported by Tan (1968). Figures 5, 6 and 7 give the corresponding axial fields for these apertures, the $\alpha = 48°$ case having been omitted because it is very little different from the results for $\alpha = 60°$. In general, the edge-current theory compares well with experiment, except for the measured intensity ripples in the axial field for $\alpha = 120°$ and 132°, which the edge-current theory is unable to account for. This is a reflection effect which occurs when the screen is inclined into the image side, and has been discussed in our previous report on the wedge-shaped screen. The E-plane aperture field measurements for $\alpha = 48°$ are believed to be unreliable because the large and rapid intensity variations in the corresponding H plane of this particular aperture appear to affect the performance of the probe.

The remarkable feature of these results is that the effect of inclining the screen into the image space ($\alpha > 90°$) produces very little change in the diffracted field in the two screens measured, except for the reflection effect. For $\alpha < 90°$ the aperture fields are affected by the proximity of the reflections of the incident wave from the screen. However, even for these cases, the field one or two wavelengths behind the aperture shows little change. It appears

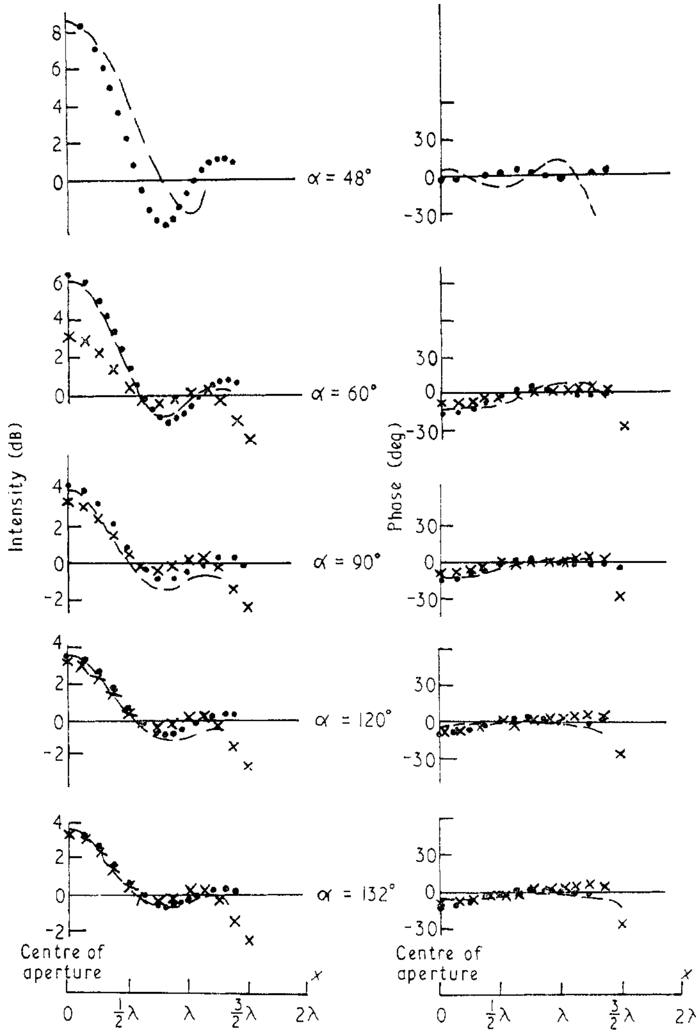

Figure 4. E-plane aperture field $E_y^2(0, y, 0)$ of circular aperture with inclination angle α, $ka = 10$ ($\lambda = \pi a/5$): – – – – experiment; • • • edge-current theory (equation (2)); × × × second Rayleigh integral (equation (17)).

therefore that the influence of screen geometry is small except very near the aperture. Extensive measurements in other parts of the diffracted field of this three-wavelength aperture, of a thirteen-wavelength aperture and also of several slit apertures support this conclusion.

From figures 3 and 4 the second Rayleigh integral gives results for the aperture field intensity and phase, whose agreement with experiment is as good as the edge-current theory for screens inclined into the image space ($\alpha > 90°$). For $\alpha < 90°$ the presence of reflected waves in front of the aperture invalidates the assumption of replacing the aperture magnetic field by the incident field, so that for these cases agreement is poor. The first Rayleigh integral predicts a constant aperture electric field equal to the incident field, which

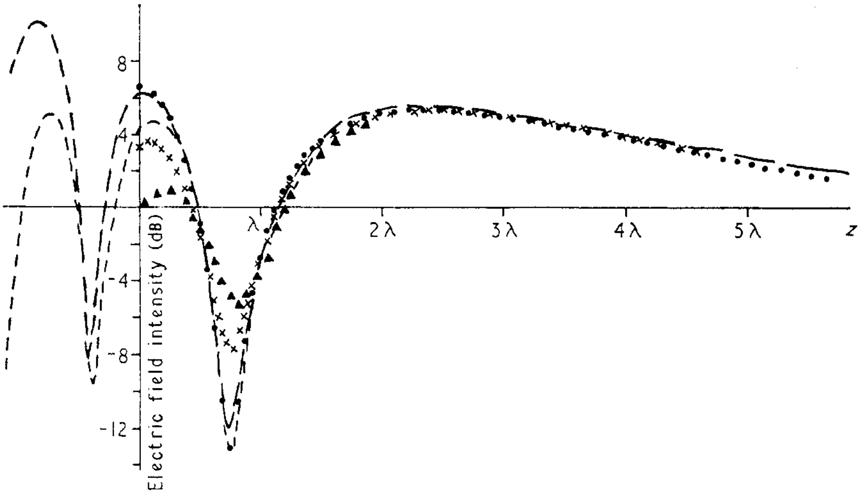

Figure 5. Axial field intensity $E_y^2(0, 0, z)$ of circular aperture, $\alpha = 60°$, $ka = 10$ ($\lambda = \pi a/5$): ——— experiment; × × × second Rayleigh integral (equation (16)); ▲ ▲ ▲ first Rayleigh integral (equation (15)); ● ● ● edge-current theory (equation (5)); - - - - $E_y^2(0, 0, z)$ for circular aperture in plane screen, $\alpha = 90°$.

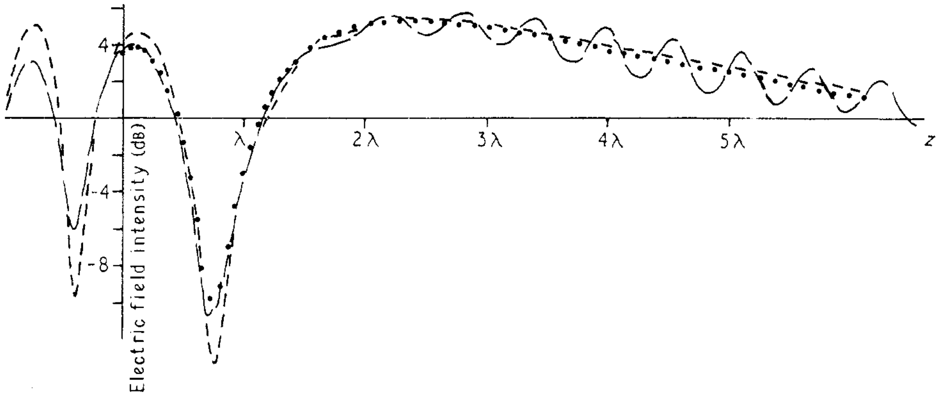

Figure 6. Axial field intensity, $E_y^2(0, 0, z)$ of circular aperture, $\alpha = 120°$, $ka = 10$: —— experiment; ● ● ● edge-currrent theory (equation (5)); - - - - $E_y^2(0, 0, z)$ of circular aperture in plane screen, $\alpha = 90°$.

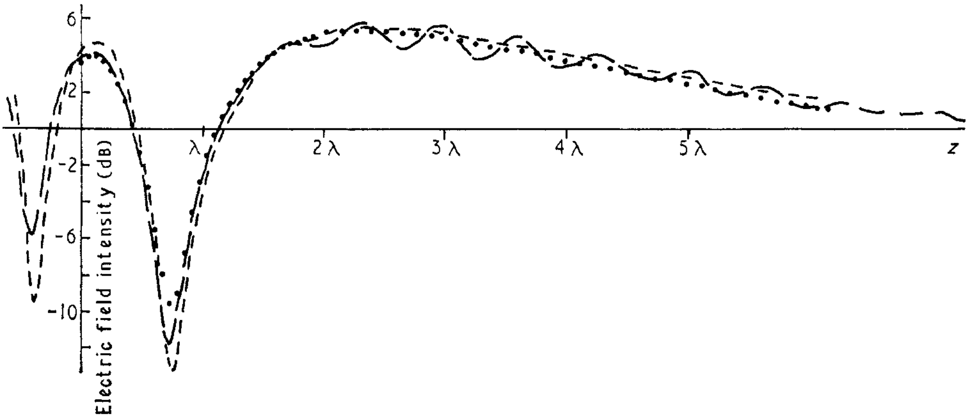

Figure 7. Axial field intensity, $E_y^2(0, 0, z)$ of circular aperture, $\alpha = 132°$, $ka = 10$: —— experiment; ● ● ● edge-current theory (equation (5)); - - - - $E_y^2(0, 0, z)$ for circular aperture in plane screen, $\alpha = 90°$.

is the assumption made in equation (13) in the first place. Figure 5 indicates that both the Rayleigh integrals practically coincide with the edge-current theory in predicting the axial field intensity for distances behind the aperture larger than one wavelength. Nearer the aperture the second Rayleigh integral is superior. Comparisons have also been made of transverse scans of the field at various planes behind the aperture, and these remarks are consistent with these results as well.

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